Sample Problem Sheet

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1. Differentiate from first principles $f(x) = \sqrt{x}$

   Solution:

$$\begin{aligned}
\frac{df}{dx} &= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(\sqrt{x+\Delta x} - \sqrt{x})(\sqrt{x+\delta x} + \sqrt{x})}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} \\
&= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x(\sqrt{x+\Delta x} + \Delta x)} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}$$

2. Differentiate the following functions:

   (a) $f(x) = 3x^2 - 1x$
   Solution: $f'(x) = 6x - 1$

   (b) $f'(x) = 5\cos(5x)$